1. Let $\vec{\mathbf{u}} = \langle 1, 2, -1 \rangle$, $\vec{\mathbf{v}} = \langle -3, 1, 5 \rangle$
   (a) Does $\vec{\mathbf{w}} = \langle 7, 0, 2 \rangle$ lie in $\operatorname{Span}\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$?

   (b) What does this tell you about the lines

$$x - 3y = 7, \quad 2x + y = 0, \quad \text{and} \quad -x + 5y = 2?$$

From Lay, Section 1.4

THEOREM 4

Let A be an $m \times n$ matrix. Then the following statements are logically equivalent. That is, for a particular A, either they are all true statements or they are all false.

a. For each $\mathbf{b}$ in $\mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.

b. Each $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A.

c. The columns of A span $\mathbb{R}^m$.

d. A has a pivot position in every row.

Answer True / False

Let
$$\vec{\mathbf{v_1}} = \langle 1, 3, 18, 2 \rangle$$
$$\vec{\mathbf{v_2}} = \langle 2, -1, 9, 0 \rangle$$
$$\vec{\mathbf{v_3}} = \langle 3, 2, -4, 1 \rangle$$
$$\vec{\mathbf{v_4}} = \langle 4, 7, 1, 3 \rangle$$
and
$$A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 1 & 4 \\ 6 & 2 & 1 \\ 5 & -17 & 32 \end{bmatrix}$$

2. The columns of A span $\mathbb{R}^4$

3. The vectors $\{\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}, \vec{\mathbf{v_3}}, \vec{\mathbf{v_4}}\}$ span $\mathbb{R}^4$

4. Let $B = [\vec{\mathbf{v_1}}\ \vec{\mathbf{v_2}}\ \vec{\mathbf{v_3}}\ \vec{\mathbf{v_4}}]$ and $\vec{\mathbf{b}} = \langle 72, -128, \pi, e^{-411} \rangle$
   The matrix equation $B\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has a unique solution

5. There exists $\vec{\mathbf{b}} \in \mathbb{R}^4$ such that $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has infinitely many solutions.