1. Let $A = \begin{bmatrix} 1 & 2 & 2 & -7 & 6 \\ 2 & 4 & 5 & -16 & 13 \\ -3 & -6 & -4 & 17 & -16 \\ 4 & 8 & 8 & -28 & 24 \end{bmatrix}$. Use that $\text{REF}(A) = \begin{bmatrix} 1 & 2 & 0 & -3 & 4 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

   (a) Give a basis for col($A$) and a basis for nul($A$).

   (b) Describe col($A$) and nul($A$) geometrically.

2. Let $\mathcal{H}$ be the subspace of $\mathbb{R}^4$ spanned by $\vec{\mathbf{v}}_1 = \begin{bmatrix} 2 \\ 4 \\ -2 \\ 8 \end{bmatrix}$, $\vec{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ 5 \\ -4 \\ 7 \end{bmatrix}$, and $\vec{\mathbf{v}}_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \end{bmatrix}$.

   Give a basis for $\mathcal{H}$ and describe $\mathcal{H}$ geometrically.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates the plane by $\frac{\pi}{3}$ counter-clockwise and projects onto the $y$-axis.
   (a) Find a basis for $\ker(T)$ and describe $\ker(T)$ geometrically
   (b) Find a basis for range$(T)$ and describe range$(T)$ geometrically

4. Let $\vec{\mathbf{u_1}} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$, $\vec{\mathbf{u_2}} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $\vec{\mathbf{u_3}} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}$, and $\vec{\mathbf{b}} = \begin{bmatrix} 17 \\ -2 \\ 4 \end{bmatrix}$
   (a) Show that the set $\mathcal{B} = \{\vec{\mathbf{u_1}}, \vec{\mathbf{u_2}}, \vec{\mathbf{u_3}}\}$ forms a basis for $\mathbb{R}^3$.
   (b) Write $\vec{\mathbf{b}}$ as a linear combination of the vectors in $\mathcal{B}$

5. True or False: If $\vec{\mathbf{v_1}} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $\vec{\mathbf{v_2}} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$, $\vec{\mathbf{v_3}} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$, then $\{\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}, \vec{\mathbf{v_3}}\}$ is a basis for $\mathbb{R}^3$