

A vector space is a nonempty set of objects  $V$ , called *vectors*, which have two operations defined: *addition of vectors* and *multiplication by scalars* (real numbers), subject to the ten axioms listed below.

The axioms must hold for all vectors  $\vec{u}, \vec{v}, \vec{w} \in V$  and for all scalars  $c$  and  $d$ .

1.  $\vec{u} + \vec{v} \in V$
2.  $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
3.  $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
4. There exists a vector  $\vec{0} \in V$  such that  $\vec{u} + \vec{0} = \vec{u}$
5. For all  $\vec{u} \in V$ , there is a vector  $-\vec{u} \in V$  such that  $\vec{u} + (-\vec{u}) = \vec{0}$
6.  $c\vec{u} \in V$
7.  $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
8.  $(c + d)\vec{u} = c\vec{u} + d\vec{u}$
9.  $c(d\vec{u}) = (cd)\vec{u}$
10.  $1\vec{u} = \vec{u}$

## Talk with the people around you for a minute

$H =$  line in the plane through the origin and  $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$  is a subspace of  $V = \mathbb{R}^2$

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Errr. . .

## Talk with the people around you for a minute

$H =$  the 1<sup>st</sup> quadrant in the plane is a subspace of  $V = \mathbb{R}^2$

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Errr. . .

## Talk with the people around you for a minute

$H =$  set of points on  $y = x^2$  is a subspace of  $V = \mathbb{R}^2$

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Errr. . .

## Talk with the people around you for a minute

Let  $\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  and  $\vec{\mathbf{v}}_2 = \begin{bmatrix} -2 \\ 1 \\ -5 \end{bmatrix}$ . Then  $H = \text{Span} \{ \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2 \}$  is a subspace of  $V = \mathbb{R}^3$

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Errr. . .

## Talk with the people around you for a minute

$H =$  the 1<sup>st</sup> and 3<sup>rd</sup> quadrant in the plane is a subspace of  $V = \mathbb{R}^2$

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Errr. . .

## Talk with the people around you for a minute

$$\text{Let } A = \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 2 & -1 \end{bmatrix}$$

Then  $H = \{ \vec{x} \in \mathbb{R}^4 \mid A\vec{x} = \vec{0} \}$  is a subspace of  $V = \mathbb{R}^4$

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Errr. . .

Let  $A = \begin{bmatrix} 2 & 6 & 2 & 8 \\ -3 & 1 & -3 & -8 \\ 3 & 4 & 3 & 10 \end{bmatrix}$

1. Fill in the blank:  $\text{nul}(A)$  is a subspace of  $\mathbb{R}^{\text{---}}$

2. Is  $\vec{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ -1 \end{bmatrix}$  in  $\text{nul}(A)$ ?

3. Find a spanning set of vectors for  $\text{nul}(A)$

4. Fill in the blank:  $\text{col}(A)$  is a subspace of  $\mathbb{R}^{\text{---}}$

5. Is  $\vec{b} = \begin{bmatrix} 44 \\ -36 \\ 51 \end{bmatrix}$  in  $\text{col}(A)$ ?

6. Find a spanning set of vectors for  $\text{col}(A)$