Recall the definition of $\det(A)$

If $A = [a_{ij}]$ is an $n \times n$ matrix, then the **determinant** of A, denoted $\det(A)$ or $|A|$, is a real number defined to by

- If $n = 1$, then $\det(A) = a_{11}$, the only entry of A
- If $n \geq 2$, then

$$\det(A) = a_{11}\det(A_{11}) - a_{12}\det(A_{12}) + \cdots + (-1)^{1+n}a_{1n}\det(A_{1n})$$

This is called the **cofactor expansion along the first row**

Note: You can expand along any row or any column, but you have to be careful with the $\pm$ signs.

Let $A = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 4 \\ 3 & 2 & 2 & 0 \\ 0 & 0 & 3 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & 2 & 0 & 4 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

1. Find $\det(A)$ by expanding along the first row
2. Find $\det(A)$ by expanding along the second column
3. Find $\det(B)$. You can pick the row or column to expand along
4. Compute $\det(AB)$ and $\det(BA)$.
   What property of determinants do your calculations demonstrate?
5. Calculate $\det(A^T)$ and $\det(B^T)$
   What property of determinants do your calculations demonstrate?

Feel free to use Mathematica's Det[ ] command for #4 and #5.

A vector space is a nonempty set of objects V, called *vectors*, which have two operations defined: *addition* of vectors and *multiplication by scalars* (real numbers), subject to the ten axioms listed below.
The axioms must hold for all vectors $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}} \in V$ and for all scalars c and d.

1. $\vec{\mathbf{u}} + \vec{\mathbf{v}} \in V$
2. $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}$
3. $(\vec{\mathbf{u}} + \vec{\mathbf{v}}) + \vec{\mathbf{w}} = \vec{\mathbf{u}} + (\vec{\mathbf{v}} + \vec{\mathbf{w}})$
4. There exists a vector $\vec{\mathbf{0}} \in V$ such that $\vec{\mathbf{u}} + \vec{\mathbf{0}} = \vec{\mathbf{u}}$
5. For all $\vec{\mathbf{u}} \in V$, there is a vector $-\vec{\mathbf{u}} \in V$ such that $\vec{\mathbf{u}} + (-\vec{\mathbf{u}}) = \vec{\mathbf{0}}$
6. $c\vec{\mathbf{u}} \in V$
7. $c(\vec{\mathbf{u}} + \vec{\mathbf{v}}) = c\vec{\mathbf{u}} + c\vec{\mathbf{v}}$
8. $(c + d)\vec{\mathbf{u}} = c\vec{\mathbf{u}} + d\vec{\mathbf{u}}$
9. $c(d\vec{\mathbf{u}}) = (cd)\vec{\mathbf{u}}$
10. $1\vec{\mathbf{u}} = \vec{\mathbf{u}}$