

Let $A = \begin{bmatrix} 1 & -3 & 0 & -1 & 3 & -3 & 0 \\ 3 & -3 & 2 & -2 & 2 & -1 & -2 \\ -3 & 3 & 3 & 2 & 1 & 0 & -2 \\ 0 & -2 & -3 & 2 & 2 & 3 & 3 \end{bmatrix}$

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1. What is $\text{rank}(A)$?
2. Find the singular value decomposition for A and verify that $A = U\Sigma V^T$
3. Find the eigenvalues $\lambda_1, \lambda_2, \dots$ of $A^T A$
4. Verify that the non-zero singular values of A appear in Σ . How many are there?
5. Verify that \vec{v}_1 , the first column of V , is an eigenvector of $A^T A$ corresponding to λ_1
6. Verify that the first column of U is $A\vec{v}_1$ normalized to have length 1