

# Shor's Algorithm to factor $n = pq$

1. Pick a random value  $a < n$
2. Compute  $\gcd(a, n)$ 
  - If  $\gcd(a, n) \neq 1$ , then we have a factor of  $n$  and we're done
  - If  $\gcd(a, n) = 1$  then continue
3. Use the quantum algorithm to find  $r = \text{ord}(a)$  in  $\mathbb{Z}_n^*$
4. If  $r$  is odd, then go to step 1 and pick another  $a$
5. If  $a^{r/2} + 1 \equiv 0 \pmod{n}$ , then go to step 1 and pick another  $a$
6. The factors of  $n$  are  $\gcd(a^{r/2} + 1, n)$  and  $\gcd(a^{r/2} - 1, n)$

Let  $n = 1\,199\,885\,077$

The goal is to factor  $n$  using Shor's algorithm

Since  $n$  is small enough, Mathematica's `MultiplicativeOrder[ ]` command can be used rather than a quantum computer :)

1. Apply Shor's algorithm with  $a = 131\,928\,655$
2. Apply Shor's algorithm with  $a = 1\,618\,912$
3. Apply Shor's algorithm with  $a = 1\,061\,873\,236$
4. What are the factors of  $n$ ?