If p is prime, how do we find $\alpha \in \mathbb{F}_p^*$ of large prime order?

Exercise 1.33 gives a reliable way to find such an α in general. This problem looks at a specific case.

1. Use Mathematica to verify that $p = 415643$ is prime.
2. Verify that $q = 207821$ is prime and that $q \mid (p-1)$. What is $\frac{p-1}{q}$?
3. What are all of the possible orders of elements in $\mathbb{F}_p^*$?
4. Let $a = 6$, and compute $a^2 \mod p$ and $a^q \mod p$.
   Use your answers to determine $\operatorname{ord}(a)$ in $\mathbb{F}_p^*$.
5. Repeat (d) with $a = 9$ and $a = 415642$.
6. Find $\operatorname{ord}(a^2)$ in $\mathbb{F}_p^*$ for the following values of a:

   $$a = 6, \quad a = 9, \quad a = 100000, \quad a = 415642$$

   Pick a few more random values of a and find $\operatorname{ord}(a^2)$ in $\mathbb{F}_p^*$.