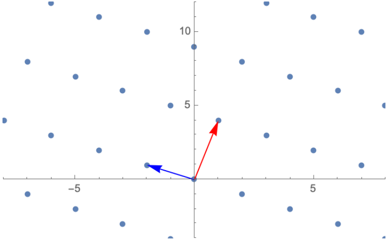
Let L be the lattice generated by $v_1 = \langle 1, 4 \rangle$ and $v_2 = \langle -2, 1 \rangle$. Thus, $\mathcal{B} = \{v_1, v_2\}$ is a basis for L.



1. Find three more points that lie on L

2. What is $\det(L)$?

3. For each set, show that each vector in the set lies on L.
   Does the set form a basis for L?
   (a) $B_1 = \{\langle 8, 5 \rangle, \langle 3, 21 \rangle\}$
   (b) $B_2 = \{\langle 64, 31 \rangle, \langle 23, 11 \rangle\}$

4. Use $\mathcal{B}$ to create a new basis $\mathcal{B}'$ for L by multiplying by several upper and lower triangular matrices.
   Verify that your set of vectors $\mathcal{B}'$ is a basis for L.

5. Consider the basis $\mathcal{B}$, your basis $\mathcal{B}'$ from #4, and any set from #3 that is a basis.
   (a) Calculate the Hadamard ratio of each basis.
   (b) What does this tell you about the skewedness of each basis?

6. Create a basis for L with a Hadamard ratio less than 0.01