

PROBLEM SET #7

Due Thursday, April 6, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

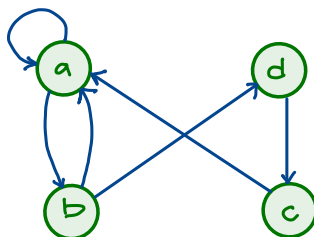
Remember to review the [Guidelines for Problem Sets](#) on the course webpage.

1. Define a relation R on \mathbb{R} by $x R y$ iff $xy > 0$.
 - (a) Give three ordered pairs in R .
 - (b) Give three ordered pairs of real numbers not in R .
 - (c) Prove that R is symmetric and transitive but not reflexive.

2. Let S be the relation defined on $\mathbb{R} \times \mathbb{R}$ by $(x, y) S (a, b)$ iff $x^2 + y^2 = a^2 + b^2$.
Prove that S is an equivalence relation and describe the equivalence classes.

3. Let T be the relation defined on $\mathbb{R} \times \mathbb{R}$ by $(x, y) T (a, b)$ iff $3x + 2y = 3a + 2b$.
Prove that T is an equivalence relation and describe the equivalence classes.

4. We can visualize a relation R on a set A by drawing a *directed graph*, or *digraph*, where the vertices are the elements of A and we draw an edge from vertex a to vertex b if $(a, b) \in R$.
For example, $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, a), (b, d), (c, a), (d, c)\}$ then the digraph is



Let $A = \{1, 2, 3, 4, 5, 6\}$ and define

$$R = \{(1, 1), (1, 6), (2, 2), (2, 3), (2, 4), (3, 3), (3, 2), (3, 4), (4, 4), (4, 2), (4, 3), (5, 5), (6, 6), (6, 1)\}$$

- (a) Draw the digraph for R .
- (b) Determine whether R is an equivalence relation on A .
- (c) In general, if S is an equivalence relation on a set B , describe the digraph for S .
Pay special attention to the equivalence classes of S .