

## PROBLEM SET #6

Due Thursday, March 30, 2023 @ 11:59 pm  
Submit as single pdf file to Canvas

Remember to review the [Guidelines for Problem Sets](#) on the course webpage.

1. Provide a proof for each statement that is true, and find a counter-example for each statement that is false. Assume that all sets are subsets of a universal set  $U$ .
  - (a) For all sets  $A$  and  $B$ , if  $A \subseteq B$  then  $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ .
  - (b) For all sets  $A$  and  $B$ ,  $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$ .
  - (c) For all sets  $A$  and  $B$ ,  $\mathcal{P}(A \times B) \subseteq \mathcal{P}(A) \times \mathcal{P}(B)$ .
  
2. Prove or give a counterexample: For all sets  $A, B$  and  $C$ , if  $A \cup C = B \cup C$  and  $A \cap C = B \cap C$ , then  $A = B$ .
  
3. Define  $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$  by  $f(x, y) = (x + 7, 2 - 3y)$ 
  - (a) Is  $f$  one-one? Prove or give a counterexample.
  - (b) Is  $f$  onto? Prove or give a counterexample.
  
4. If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  are functions and  $g \circ f$  is onto, must  $f$  be onto? Prove or give a counterexample.