PROBLEM SET #5

Due Thursday, March 2, 2023 @ 11:59 pm Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Let $x \in \mathbb{R}$. Prove x is irrational iff $\frac{1}{x}$ is irrational.

- 2. Prove 3 $| (n^3 7n + 3) \forall$ integers $n \ge 0$.
- 3. Prove $\sqrt{n} < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} \quad \forall n \in \mathbb{Z}, n \ge 2.$

Hint: At some point in your proof, it might be useful to remember that $\sqrt{k} < \sqrt{k+1}$ $\forall k \in \mathbb{N}$.

4. Consider the sequence defined by

$$a_0 = 12$$
, $a_1 = 29$, and $a_k = 5a_{k-1} - 6a_{k-2}$, $\forall k \in \mathbb{Z}, k \ge 2$

- (a) Compute the first six terms of the sequence (through a_5).
- (b) Prove that $a_n = 5 \cdot 3^n + 7 \cdot 2^n \quad \forall n \in \mathbb{Z}, n \ge 0.$

References for problems: 2. Epp, *Discrete Mathematics with Applications, 4th edition*, Exercise 5.3.10; 3. Epp, Exercise 5.3.21; 4. Epp, Exercises 5.4.5