Problem Set #4

Due Thursday, February 23, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage.

1. Prove the following by proving the contrapositive using two cases.

   $\forall m, n \in \mathbb{Z}$, if nm is odd, then n is odd and m is odd.

2. Prove that there is no least positive rational number.

3. For all integers n, prove n^3 is even iff n is even.

4. Determine whether each statement is true or false.
   If it is true, then give a proof. If it is false, then provide a counterexample.

   (a) The difference of the squares of any two consecutive integers is odd.

   (b) For all integers n, if n is prime, then $(-1)^n = -1$.

   (c) The sum of any four consecutive integers has the form $4k + 2$ for some integer k.

5. Use mathematical induction to prove that

$$1 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{for all integers } n \geq 1$$

References for problems: 1. Ernst, *Introduction to Proof via Inquiry-Based Learning*, Section 2.5; 2. Epp, *Discrete Mathematics with Applications, 4th edition*, Exercise 4.6.7; 4. Epp, Exercises 4.1.58, 4.1.51, 4.4.37; 5. Epp, Exercises 5.1.10