

PROBLEM SET #3

Due Thursday, February 16, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the [Guidelines for Problem Sets](#) on the course webpage.

1. Consider the statement $\exists x$ s.t. $x > x^3$.
Give a domain where the statement is true and another where the statement is false. Explain.
2. For each statement,
 - (i) Write it as an English sentence that does not use symbols for quantifiers;
 - (ii) Write the negation in symbolic form;
 - (iii) Write the negation as an English sentence that does not use symbols for quantifiers;
 - (iv) Give the truth value of the statement and its negation (you do not need to prove these claims).
 - (a) $\exists m \in \mathbb{Z}$ s.t. $\forall n \in \mathbb{Z}, m > n$
 - (b) $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}$ s.t. $m^2 > n$
3. Consider $\forall y \exists x \exists z$ s.t. $T(x, y, z) \wedge Q(x, y)$
 - (a) Suppose that
the domain of x is the set of math classes offered at Wheaton,
the domain of y is the set of students in Discrete Math, and
the domain of z is the set of all math professors at Wheaton.
Let $T(x, y, z)$ be the predicate “ y took class x with z ” and
 $Q(x, y)$ be the predicate “ y loves the class x ”.
What does the original statement say in words?
 - (b) Negate the statement so that all negation symbols immediately precede predicates.
Be sure to show all your steps!
 - (c) Using the same definitions as in (a), what does the negation of the statement mean in words?
4. Are the following statements true or false? Give a proof for your conclusions.
 - (a) If a, b , and c are integers, then $ab + ac$ is an even integer.
 - (b) If b and c are odd integers and a is an integer, then $ab + ac$ is an even integer.
5. Prove that for all integers a and m , if a and m are the legs of a right triangle with hypotenuse $m + 1$, then a is an odd integer.

References for problems: 1,3. Rachele DeCoste; 2. Sundstrom, *Mathematical Reasoning Writing and Proof, Version 3*, Exercises 2.4.4 & 2.4.5; 4. Sundstrom, Exercise 1.2.7