

PROBLEM SET #2

Due Thursday, February 9, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the [Guidelines for Problem Sets](#) on the course webpage.

1. (a) Use a truth table to show that $p \rightarrow q \equiv (\sim p \vee q)$
 (b) Use DeMorgan's Law and part (a) to simplify $\sim (p \rightarrow q)$
 (c) Use DeMorgan's Law and part (a) to simplify $\sim ((\sim (p \wedge q)) \rightarrow (r \wedge s))$

2. (a) Show that the following statements are all logically equivalent

$$p \rightarrow (q \vee r), \quad (p \wedge \sim q) \rightarrow r, \quad (p \wedge \sim r) \rightarrow q$$

- (b) Use the logical equivalences from part (a) to rewrite the following sentence in two different ways.
 If n is prime, then n is odd or $n = 2$.

3. Legal Seafoods restaurants use the tagline "If it isn't fresh, it isn't Legal".

- (a) Write the converse of the tagline. Do you think this is true?
- (b) Write the contrapositive of the tagline. Do you think this is true?
- (c) Use 1(b) to write the negation of the tagline.
- (d) Suggest a logically equivalent tagline that avoids the use of "isn't" and "not".

4. Let $Q(n)$ be the predicate " $n^2 \leq 30$ "

- (a) Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$, and indicate which of these statements are true, and which are false.
- (b) Find the truth set of $Q(n)$ if the domain of n is \mathbb{Z} .
- (c) Find the truth set of $Q(n)$ if the domain of n is \mathbb{N} .

5. Let the domain of x be \mathbb{R} , and let

$\mathcal{I}(x)$ be "x is an integer",
 $\mathcal{N}(x)$ be "x is a natural number", and
 $\mathcal{P}(x)$ be "x is positive"

Rewrite each statement without using quantifiers or variables, and indicate which statements are true and which are false. Be sure to explain your answers.

- (a) $\forall x, \mathcal{I}(x) \rightarrow \mathcal{N}(x)$
- (b) $\forall x, (\mathcal{I}(x) \wedge \mathcal{P}(x)) \rightarrow \mathcal{N}(x)$
- (c) $\exists x$ such that $\mathcal{I}(x) \wedge \sim \mathcal{N}(x)$

References for problems: 2. Epp, *Discrete Mathematics with Applications, 4th edition*, Exercise 2.2.14; 4. Epp, Exercise 3.1.4; 5. Epp, Exercise 3.1.28