

## Discuss with your partner(s)

1. Let  $f: \mathbb{Z} \rightarrow \mathbb{Z}$  where  $f(k) = k + 7$ ,  
 $g: \mathbb{Z} \rightarrow \{0, 1, 2, 3, 4\}$  where  $g(k) = k + 1 \pmod{5}$ ,  
and  $h: \{0, 1, 2, 3, 4\} \rightarrow \{0, 1, 2, 3, 4\}$  where  $h(a) = a^2 \pmod{5}$   
Determine if the following expressions are defined. If so, find the value.

$$(g \circ f)(2), \quad (h \circ g)(2), \quad (h \circ f)(4), \quad (h \circ f)(-3), \quad (h \circ g \circ f)(0)$$

2. If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are functions and  $g \circ f$  is one-to-one, must  $g$  be one-to-one? Prove or give a counterexample.
3. If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are functions and  $g \circ f$  is one-to-one, must  $f$  be one-to-one? Prove or give a counterexample.
4. Let  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $g(x) = 2x - 7$ . Prove that  $g$  is one-one and onto.