

Work on these with your partner(s) at the board

1. Consider the sum
$$\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \cdots + \frac{1}{n(n+1)}$$

- (a) Compute the sum for a few values of n
- (b) Form a conjecture for the value of the sum that depends only on n
- (c) Use induction to prove your conjecture

2. Determine whether each statement is true or false.

If it is true, then give a proof. If it is false, then provide a counterexample.

- (a) $\forall n \in \mathbb{N}$, $6n^2 + 1$ is not a perfect square
- (b) $\forall n \in \mathbb{N}$, $23n^2 + 1$ is not a perfect square
- (c) $\forall n \in \mathbb{N}$, $991n^2 + 1$ is not a perfect square