Work on these with your partner(s) at the board

Prove each of the following. Explicitly list the type of proof used.

1. $\forall a \in \mathbb{Q},\ \exists b \in \mathbb{Q}$ s.t. $a^b \in \mathbb{Q}$
2. $\forall n \in \mathbb{N}$ with $n \leq 4,\ (n+1)^3 \geq 3n$
3. The sum of an even integer and an odd integer is an odd integer.
4. The square of an odd number is odd.
5. The sum any rational number and any irrational number is irrational.
6. The square of any odd integer has the form $8m + 1$ for some integer m.
7. $\forall n \in \mathbb{Z}, n^2 \geq n$
8. Given any two consecutive integers, one is even and one is odd.

Based on a worksheet from Rachelle DeCoste