Some Big Ideas, Week 9

Mar 27 – Mar 31, 2023

- **Definition**: Let A and B be sets. A **relation R from A to B** is a subset of $A \times B$.

  If $(a, b) \in A \times B$, we say **a is related to b by R**, denoted aRb, iff $(a, b) \in R$.

  A is the **domain** of R, and B is the **codomain** of R.

- **Note**: Any function $f : A \to B$ defines a relation R by aRb iff $b = f(a)$.

- **Definition**: A **relation on a set A** is a relation from A to A.

- **Definition**: Let R be a relation on a set A.
  - R is **reflexive** iff for all $a \in A$, aRa,
    or equivalently, for all $a \in A$, $(a, a) \in R$.
  - R is **symmetric** iff for all $a, b \in A$, if aRb then bRa,
    or equivalently, for all $a, b \in A$, if $(a, b) \in R$ then $(b, a) \in R$.
  - R is **transitive** iff for all $a, b, c \in A$, if aRb and bRc then aRc,
    or equivalently, for all $a, b, c \in A$, if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

- **Definition**: Let A be a set and R a relation on A. Then R is an **equivalence relation** iff R is reflexive, symmetric, and transitive.

- **Definition**: Let A be a set and R an equivalence relation on A. For each element $a \in A$, define the **equivalence class of a**, denoted $[a]$, to be the set of elements in A that are related to a:

$$[a] = \left\{ b \in A \;\middle|\; aRb \right\}$$

- **Definition**: A **partition** of a set A is collection of non-empty, mutually disjoint subsets of A such that every element of A is in exactly one of the subsets.

  For example, if E denotes the even integers and O denotes the odd integers, then a partition of $\mathbb{Z}$ is $\{E, O\}$.

- **Theorem** (8.3.4, Epp pg 469): If A is a set and R is a relation on A, then the distinct equivalence classes of R form a partition of A.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the Tentative Weekly Syllabus for the specific sections relevant for this week.