Some Big Ideas, Week 5

Feb 20 – Feb 24, 2023

⊙ **Principle of Mathematical Induction:**

Let $P(n)$ be a property that is defined for integers n. Let a be a fixed integer.

Suppose the following two statements are true:

1. $P(a)$ is true.
2. $\forall k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.

Then $\forall n \geq a, P(n)$ is true.

⊙ **General structure of a Proof by Induction:**

Start by giving the statement that you want to prove:

Let $P(n)$ be the statement ...

To prove $P(n)$ is true for all $n \geq a$, requires two steps:

1. **Base case:** Prove that $P(a)$ is true.
2. **Inductive case:** Assume that $P(k)$ is true, and prove that $P(k+1)$ is true.

   "$P(k)$ is true" is called the **inductive hypothesis**.

If you successfully proved both results, then you can conclude

Thus, by the principle of mathematical induction, $P(n)$ is true $\forall n \geq a$.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the Tentative Weekly Syllabus for the specific sections relevant for this week.