

Some Big Ideas, Week 3

Feb 6 – Feb 10, 2023

- ⊙ **Definition:** A **predicate** is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables.

The **domain** of a predicate variable is the set of all values that may be substituted in place of the variable.

If $P(x)$ is a predicate and x has domain D , the **truth set** of $P(x)$ is the set of all elements of D that make $P(x)$ true when they are substituted for x :

$$\{x \in D \mid P(x)\}$$

- ⊙ **Definition:** The **universal quantifier** \forall means “for all”:

$$\forall x \in D, P(x) \text{ means } P(x) \text{ is true for every } x \in D$$

A universal statement is false if $P(x)$ is false for at least one $x \in D$.

- ⊙ **Definition:** The **existential quantifier** \exists means “there exists”:

$$\exists x \in D \text{ such that } P(x) \text{ means } P(x) \text{ is true for at least one } x \in D$$

An existential statement is false if $P(x)$ is false for every $x \in D$.

- ⊙ The negation of a universal statement “ $\forall x \in D, P(x)$ ” is logically equivalent to the statement “ $\exists x \in D$ such that $\sim P(x)$ ”.
- ⊙ The negation of an existential statement “ $\exists x \in D$ such that $P(x)$ ” is logically equivalent to the statement “ $\forall x \in D, \sim P(x)$ ”.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.