

Some Big Ideas, Week 2

Jan 30 – Feb 3, 2023

These are some of the Big Ideas that we will discuss this week. The sections from the optional texts listed on the Weekly Syllabus on the course website provide more background and fill in additional details.

- ⊙ **Definition:** A **proposition** or **statement** is a sentence that is true or false, but not both.
- ⊙ **Notation:** If p and q are propositions, then
 - $\sim p$ or $\neg p$ means “not p ”, also called the **negation** of p .
 - $p \vee q$ means “ p or q ”, also called the **disjunction** of p and q .
 - $p \wedge q$ means “ p and q ”, also called the **conjunction** of p and q .
 - $p \rightarrow q$ means “ p implies q ”, also called the **conditional**. p is the **hypothesis** and q is the **conclusion**
- ⊙ **Definition:** The **truth table** for a logical expression displays the truth values for the expression corresponding to all possible combinations of truth values for the propositions in the expression.
- ⊙ **Notation:** If expressions P and Q are logically equivalent, then we denote this with $P \equiv Q$.
- ⊙ **DeMorgan’s Laws:** If P and Q are logical expressions, then
 - $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
 - $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
- ⊙ **Definition:** The **biconditional** “ p if-and-only-if q ” means $(p \rightarrow q) \wedge (q \rightarrow p)$. We will often abbreviate to “ p iff q ”.
- ⊙ **Definition:** The **contrapositive** of a conditional statement “If p then q ” is the statement “If $\neg q$ then $\neg p$ ”.
- ⊙ **Definition:** The **converse** of a conditional statement “If p then q ” is the statement “If q then p ”.
- ⊙ **Definition:** The **inverse** of a conditional statement “If p then q ” is the statement “If $\neg p$ then $\neg q$ ”.

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.