Some Big Ideas, Week 10

Apr 3 – Apr 7, 2023

- **Definition:** A **sample space** is the set of all possible outcomes of an experiment or random process. An **event** is a subset of a sample space.

- **Definition:** If S is a finite sample space where all outcomes are equally likely and E is an event in S, then the **probability of E** is

$$P(E) = \frac{\text{number of outcomes in } E}{\text{total number of outcomes in } S} = \frac{|E|}{|S|}$$

  Note: Epp uses the notation $N(A)$ for the cardinality of a finite set rather than $|A|$.

- **Additive Principle:** If an event E can occur in m ways and the event F can occur in n ways where E and F are disjoint, then the event "E or F" can occur in $m + n$ ways.

- **Multiplicative Principle:** If an event E can occur in m ways, and each possibility for E allows for exactly n ways for the event F to occur, then the event "E and F" can occur in $m \cdot n$ ways.

- **Definition:** A **permutation** of a set A is an ordered, non-repetitive arrangement of all elements of A. For example, if $A = \{a, b, c\}$, then there are six permutations of A: $abc, acb, cab, cba, bca, bac$

- **Definition:** A **k-permutation** of A an ordered, non-repetitive arrangement of k elements of A. For example, if $A = \{a, b, c, d\}$, a 2-permutation of A is bc and a 3-permutation is cad.

- **Theorem (Epp 9.2.3):** The number of k-permutations from a set with n elements, denoted $P(n, k)$, is

$$P(n,k) = \frac{n!}{(n-k)!}$$

- If A and B are finite sets, then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Some of the resources I used in constructing the Big Ideas notes this semester are: Ernst: *Introduction to Proof via Inquiry-Based Learning*; Epp: *Discrete Mathematics with Applications, 4th edition*; Levin: *Discrete Mathematics, An Open Introduction, 3rd edition*; Sundstrom: *Mathematical Reasoning, Writing and Proof, Version 3*; and the notes of my colleague, Rachelle DeCoste at Wheaton.

Check the Tentative Weekly Syllabus for the specific sections relevant for this week.