Problem Set #7

Due Thursday, November 30, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage when writing up the solutions with your group, and don't forget to submit the Partner Evaluation through Canvas.

1. Let W be the subspace of $\mathbb{R}^3$ spanned by $\vec{\mathbf{u}}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ and $\vec{\mathbf{u}}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$ and let $\vec{\mathbf{y}} = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$.
   (a) Verify that $\{\vec{\mathbf{u}}_1, \vec{\mathbf{u}}_2\}$ is an orthogonal basis for W.
   (b) Use the Orthogonal Decomposition Theorem to write $\vec{\mathbf{y}} = \hat{y} + \vec{\mathbf{z}}$ where $\hat{y} \in W$ and $\vec{\mathbf{z}} \in W^\perp$.

2. Let $A = \begin{bmatrix} 4 & -2 & -6 & 4 \\ -4 & 8 & -1 & 0 \\ -7 & -2 & 10 & -8 \\ 3 & -6 & 0 & -4 \\ 3 & 4 & -8 & 7 \\ 10 & -7 & -2 & 10 \\ 6 & 10 & -2 & -1 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 4 \\ -1 \\ 1 \\ 10 \\ -4 \\ -4 \\ -6 \end{bmatrix}$
   (a) Show that the system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is inconsistent.
   (b) Find the least squares solution $A\hat{x} = \hat{b}$.

3. Let $A = \begin{bmatrix} 42 & -3 & 24 & -6 \\ -30 & 33 & 168 & -42 \\ 236 & -4 & 131 & -53 \\ 1064 & -40 & 500 & -206 \end{bmatrix}$
   (a) Find the eigenvalues of A. Using Mathematica is fine.
   (b) Use your answer from (a) to explain how you know A is diagonalizable.
   (c) Diagonalize A. That is, write $A = PDP^{-1}$ where D is diagonal.

4. Let $A = \begin{bmatrix} \frac{733}{325} & -\frac{2}{13} & -\frac{306}{325} \\ -\frac{2}{13} & \frac{53}{26} & \frac{3}{26} \\ -\frac{306}{325} & \frac{3}{26} & \frac{1109}{650} \end{bmatrix}$
   (a) Explain how you know that A is orthogonally diagonalizable.
   (b) Orthogonally diagonalize A. That is, write $A = PDP^T$ where D is diagonal.