Problem Set #5

Due Thursday, October 26, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage when writing up the solutions with your group, and don't forget to submit the Partner Evaluation through Canvas.

1. Let $A = \begin{bmatrix} -6 & 24 & 47 \\ 12 & 8 & -45 \\ -12 & -24 & 31 \end{bmatrix}$

   (a) Give a basis for $\mathrm{nul}(A)$ and describe $\mathrm{nul}(A)$ geometrically.

   (b) Give a basis for $\mathrm{col}(A)$ and describe $\mathrm{col}(A)$ geometrically.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ where $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$.

   (a) Find a basis for $\ker(T)$ and describe $\ker(T)$ geometrically.

   (b) Find a basis for $\mathrm{range}(T)$ and describe $\mathrm{range}(T)$ geometrically.

   (c) Describe T in geometric terms.
   e.g. "T rotates the plane by $\frac{\pi}{3}$ radians counter-clockwise", or "T projects the plane onto the x-axis", etc.
   To be clear, T doesn't do either of these, but these are examples of how you can describe T.
   It may be useful to pick a few specific points in $\mathbb{R}^2$ and see what their image is under T.

3. Show that $\mathcal{B} = \left\{4 - 31t - 7t^2,\ 1 + t,\ 5t + t^2\right\}$ is *not* a basis for $\mathbb{P}_2$.

4. (a) Let $S = \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_k\}$ be a set of vectors in $\mathbb{R}^n$ with $k < n$. Use a theorem from earlier in the semester to explain why S cannot be a basis for $\mathbb{R}^n$.

   (b) Let $S = \{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_k\}$ be a set of vectors in $\mathbb{R}^n$ with $k > n$. Use a theorem from earlier in the semester to explain why S cannot be a basis for $\mathbb{R}^n$.

   (These problems are essentially the same as Exercises 4.3.29 and 4.3.30 from the text, Lay's Linear Algebra, 4th edition)