Problem Set #4

Due Thursday, October 19, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage when writing up the solutions with your group, and don't forget to submit the Partner Evaluation through Canvas.

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates the plane by $\frac{\pi}{3}$ radians clockwise about the point $(3, -4)$.
   (a) Explain why T is not a *linear* transformation.
   (b) Find the 3×3 matrix A that produces T using homogeneous coordinates.
   (c) What is the image of the point $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ under T?

2. Let $A = \begin{bmatrix} 0 & 3 & 2 & 1 \\ 1 & 3 & 0 & 2 \\ 5 & 4 & 0 & 3 \\ 3 & 0 & 1 & 2 \end{bmatrix}$
   (a) Compute $\det(A)$ by hand using a cofactor expansion.
   You may pick which row or column you expand along.
   (b) Show that A is invertible.
   (c) What is $\det(A^{-1})$? Note that you don't need to actually find A^{-1}!

3. Let $A = \begin{bmatrix} -5 & 3 & 9 & 8 & 2 & 11 & -2 & 10 \\ -11 & 7 & 0 & 8 & 0 & -8 & 6 & -9 \\ -2 & 12 & 0 & 7 & 9 & 12 & 1 & -5 \\ -2 & 3 & 3 & 6 & 4 & 4 & -3 & -6 \\ 9 & 5 & -1 & -11 & -9 & 2 & -6 & 10 \\ 9 & 5 & -6 & 0 & -11 & 7 & -8 & -11 \\ 0 & -2 & 12 & -3 & 2 & 7 & 1 & -9 \\ -5 & -8 & -5 & 4 & 10 & -3 & -11 & 9 \end{bmatrix}$
   Notice that A is given in Mathematica format on the Problem Sets page so that you can copy and paste into your Mathematica notebook.
   (a) What is the determinant of A? Please don't do this by hand!
   (b) Is A invertible? Why or why not?
   (c) Is the linear transformation $T : \mathbb{R}^8 \to \mathbb{R}^8$ defined by $T(\vec{\mathbf{x}}) = A\vec{\mathbf{x}}$ one-one? Explain.

4. Let $H = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \mid a, b \leq 0 \right\}$, the 3rd quadrant in $\mathbb{R}^2$.
   (a) Is H closed under vector addition? Explain.
   (b) Is H closed under scalar multiplication? Explain.
   (c) Is H a subspace of $\mathbb{R}^2$? Explain.

5. Let $B = \begin{bmatrix} 3 & 6 & 15 \\ 6 & 12 & 29 \\ 3 & 6 & 13 \end{bmatrix}$
   (a) Give a non-zero vector in $\text{nul}(B)$. Explain.
   (b) Give a non-zero vector in $\text{col}(B)$. Explain.
   (c) Is $\vec{\mathbf{b}} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix}$ in $\text{col}(B)$? Explain.
   (d) Give a vector in $\mathbb{R}^3$ that is **not** in $\text{col}(B)$. Explain.