Problem Set #2

Due Thursday, September 14, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage when writing up the solutions with your group, and don't forget to submit the Partner Evaluation through Canvas.

1. Consider the augmented matrix $\left[\begin{array}{ccc|c} 44 & 89 & 6 & -357 \\ -16 & -32 & -2 & 130 \\ 10 & 21 & 2 & -80 \end{array}\right]$

   (a) This augmented matrix corresponds to a system of linear equations in three variables. What is the system of equations?

   (b) This augmented matrix corresponds to a vector equations in three variables. What is the vector equation?

   (c) This augmented matrix corresponds to a matrix equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$. What are A and $\vec{\mathbf{b}}$?

   (d) Solve the system, and give your answer as a solution to the system from part (a).

2. Let $A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -6 & 1 & -7 \\ 3 & 9 & -4 & 18 \\ 1 & 3 & 1 & -1 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} 7 \\ -23 \\ 57 \\ -2 \end{bmatrix}$.

   (a) Write the general solution to $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ in parametric form.

   (b) Are the columns of A linear independent or linearly dependent? Explain.

   (c) Do the columns of A span $\mathbb{R}^4$? Explain.

   (d) Does $\vec{\mathbf{b}}$ lie in the span of the columns of A? Explain.

3. Each statement is either true (in all cases) or false (for at least one example). If false, construct a specific counterexample to show that the statement is not always true. If a statement is true, give a justification. (One specific example cannot explain why a statement is always true.)

   (a) The columns of every 3×5 matrix A are linearly dependent.

   (b) If $\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3$ are in $\mathbb{R}^3$ and $\vec{\mathbf{v}}_3$ is *not* a linear combination of $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ then $\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{v}}_3\}$ is linearly independent.

   (c) If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are linear independent and $\vec{\mathbf{w}}$ lies in Span $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$, then $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}\}$ is linearly dependent.

   (The problem is very similar to Exercises 1.7.33-38 from the text, Lay's Linear Algebra, 4th edition)