## Problem Set \#1

Due Thursday, September 7, 2023 @ 11:59 pm
Submit as single pdf file to Canvas

Remember to review the Guidelines for Problem Sets on the course webpage when writing up the solutions with your group, and don't forget to submit the Partner Evaluation through Canvas.

1. Find the general solution of the system whose augmented matrix is

$$
\left[\begin{array}{cccc|c}
-1 & -3 & -10 & -1 & -14 \\
-2 & 1 & 1 & 0 & 1 \\
2 & 4 & 14 & 1 & 21
\end{array}\right]
$$

For this problem, perform the row reduction by hand and document each step in the process.
2. Find an equation involving $g, h$, and $k$ that makes this augmented matrix correspond to a consistent system:

$$
\left[\begin{array}{ccc|c}
1 & 3 & -5 & g \\
0 & 2 & -1 & h \\
-2 & 4 & 5 & k
\end{array}\right]
$$

(The problem is very similar to Exercise 1.1.25 from the text, Lay's Linear Algebra, 4th edition)
3. Suppose a $4 \times 6$ coefficient matrix for a system has four pivot columns. Is the system consistent? Why or why not?
(The problem is essentially the same as Exercise 1.1.26 from the text, Lay's Linear Algebra, 4th edition)
4. In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

| Velocity $(100 \mathrm{ft} / \mathrm{sec})$ | 0 | 2 | 4 | 6 | 8 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Force $(100 \mathrm{lb})$ | 0 | 2.90 | 14.8 | 39.6 | 74.3 | 119 |

(a) Set up the system of six equations in six unknowns to find the interpolating polynomial of degree five for these data, $p(t)=a_{0}+a_{1} t+a_{2} t^{2}+a_{3} t^{3}+a_{4} t^{4}+a_{5} t^{5}$.
(b) What is the polynomial $p(t)$ ? You can use technology to perform the row reduction.
(c) Graph the points and your polynomial on the same axes to verify that the polynomial goes through all of the points.
(d) Use your polynomial to estimate the force when the projectile is moving at $750 \mathrm{ft} / \mathrm{sec}$.
(e) What happens if you try to use a cubic polynomial rather than a polynomial of degree 5 ?
(The problem is essentially the same as Exercise 1.2.34 from the text, Lay's Linear Algebra, 4th edition. It may be useful to read the description before exercise 1.2.33.)
5. Determine if $\overrightarrow{\mathbf{b}}$ is a linear combination of the vectors formed from the columns of the matrix $A$.

$$
A=\left[\begin{array}{ccc}
1 & 0 & 2 \\
-3 & 1 & 1 \\
1 & 4 & 5
\end{array}\right], \quad \overrightarrow{\mathbf{b}}=\left[\begin{array}{c}
16 \\
-3 \\
21
\end{array}\right]
$$

You can use technology for the computations.
(The problem is very similar to Exercise 1.3.14 from the text, Lay's Linear Algebra, 4th edition)

