From Lay, Section 2.1

THEOREM 2

Let A be an $m \times n$ matrix, and let B and C have sizes for which the indicated sums and products are defined.

a. $A(BC) = (AB)C$ (associative law of multiplication)

b. $A(B + C) = AB + AC$ (left distributive law)

c. $(B + C)A = BA + CA$ (right distributive law)

d. $r(AB) = (rA)B = A(rB)$ for any scalar r

e. $I_m A = A = AI_n$ (identity for matrix multiplication)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -4 \\ 3 & -6 \end{bmatrix}$, and $D = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & 3 \end{bmatrix}$

1. Compute AB and BA by hand
2. Compute AC and BC by hand
3. Compute AD and DA by hand
4. What interesting properties of matrix multiplication do these examples demonstrate?

Let $F = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 3 \\ 0 & 4 & -7 \end{bmatrix}$, $G = \begin{bmatrix} 3 & -2 & 4 \\ 7 & 12 & 8 \\ 16 & 2 & 3 \end{bmatrix}$, $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Feel free to use today's Mathematica notebook for these computations.

5. Find F^{-1} and G^{-1}
6. Compare the following products: $F^{-1}G^{-1}, \quad G^{-1}F^{-1}, \quad (FG)^{-1}, \quad (GF)^{-1}$
7. Compare $(FG)^T$, F^TG^T, and G^TF^T
8. Find $\left(F^T\right)^{-1}$
9. Compare F, E_1F, and E_2E_1F

What general observations can you make from your computations?