1. Let $A = \begin{bmatrix} 1 & 24 & -13 & -12 \\ 1 & 3 & -2 & -1 \\ 7 & 0 & -3 & 4 \end{bmatrix}$. Find bases for $\text{col}(A)$, $\text{nul}(A)$, and $\text{row}(A)$.

2. If A is 6×11 of rank 4, what is the dimension of $\text{nul}(A)$?

3. If A is the matrix corresponding to a one-one linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^8$, what is the dimension of $\text{nul}(A)$? of $\text{row}(A)$? of $\text{nul}(A^T)$?

A motivating example for Markov Chains

A town recently added a new high speed internet service provider so that it now has three ISPs: A, C, and V Each ISP runs promotions to entice customers to switch to their service, and the effects over the last year has been:

15% of the A customers switch to C and 10% switch to V
15% of the C customers switch to A and 5% switch to V
5% of the V customers switch to A and 10% switch to C

Assume that these trends continue.

If A currently has 50% of the customers, C has 30% and V has 20%, what will the distribution of customers be after 1 year? 3 years? 10 years? 20 years?