From Lay, Section 4.3

THEOREM 5

The Spanning Set Theorem

Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be a set in V, and let $H = \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$.

a. If one of the vectors in S—say, $\mathbf{v}_k$—is a linear combination of the remaining vectors in S, then the set formed from S by removing $\mathbf{v}_k$ still spans H.

b. If $H \neq \{\mathbf{0}\}$, some subset of S is a basis for H.

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👋 Justification:

a. We are given $\vec{\mathbf{v_k}} = a_1\vec{\mathbf{v_1}} + a_2\vec{\mathbf{v_2}} + \cdots + a_{k-1}\vec{\mathbf{v_{k-1}}} + a_{k+1}\vec{\mathbf{v_{k+1}}} + \cdots + a_p\vec{\mathbf{v_p}}$

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Let $\vec{\mathbf{u}} \in \text{Span}\left\{\vec{\mathbf{v_1}}, \ldots, \vec{\mathbf{v_p}}\right\}$ so that

$$\begin{aligned}\vec{\mathbf{u}} &= c_1\vec{\mathbf{c_1}} + \cdots + c_k\vec{\mathbf{v_k}} + \cdots + c_p\vec{\mathbf{v_p}} \\ &= c_1\vec{\mathbf{c_1}} + \cdots + c_k\left(a_1\vec{\mathbf{v_1}} + a_2\vec{\mathbf{v_2}} + \cdots + a_{k-1}\vec{\mathbf{v_{k-1}}} + a_{k+1}\vec{\mathbf{v_{k+1}}} + \cdots + a_p\vec{\mathbf{v_p}}\right) + \cdots + c_p\vec{\mathbf{v_p}}\end{aligned}$$

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Thus, $\vec{\mathbf{u}} \in \text{Span}\left\{\vec{\mathbf{v_1}}, \ldots, \vec{\mathbf{v_{k-1}}}, \vec{\mathbf{v_{k+1}}}, \vec{\mathbf{v_p}}\right\} \quad \Rightarrow \quad \text{Span}\, S = \text{Span}\,\{S - \vec{\mathbf{v_k}}\}$

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We haven't changed the span, so the remaining set must be a basis.

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The condition $H \neq \left\{\vec{0}\right\}$ is a technicality since the subspace $H = \left\{\vec{0}\right\}$ has no basis:

$H = \text{Span}\left\{\vec{0}\right\}$, but $\left\{\vec{0}\right\}$ is not a linearly independent set