

Recall the definition of $\det(A)$

If $A = [a_{ij}]$ is an $n \times n$ matrix, then the **determinant** of A , denoted $\det(A)$ or $|A|$, is a real number defined to be

- If $n = 1$, then $\det(A) = a_{11}$, the only entry of A
- If $n \geq 2$, then

$$\det(A) = a_{11} \det(A_{11}) - a_{12} \det(A_{12}) + \cdots + (-1)^{1+n} a_{1n} \det(A_{1n})$$

This is called the **cofactor expansion along the first row**

Note: You can expand along any row or any column, but you have to be careful with the \pm signs.

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 4 \\ 3 & 2 & 2 & 0 \\ 0 & 0 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 0 & 4 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

1. Find $\det(A)$ by expanding along the first row
2. Find $\det(A)$ by expanding along the second column
3. Find $\det(B)$. You can pick the row or column to expand along
4. Compute $\det(AB)$ and $\det(BA)$.
What property of determinants do your calculations demonstrate?
5. Calculate $\det(A^T)$ and $\det(B^T)$
What property of determinants do your calculations demonstrate?

Feel free to use Mathematica's `Det[]` command for #4 and #5.

A vector space is a nonempty set of objects V , called *vectors*, which have two operations defined: *addition of vectors* and *multiplication by scalars* (real numbers), subject to the ten axioms listed below.

The axioms must hold for all vectors $\vec{u}, \vec{v}, \vec{w} \in V$ and for all scalars c and d .

1. $\vec{u} + \vec{v} \in V$
2. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
3. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
4. There exists a vector $\vec{0} \in V$ such that $\vec{u} + \vec{0} = \vec{u}$
5. For all $\vec{u} \in V$, there is a vector $-\vec{u} \in V$ such that $\vec{u} + (-\vec{u}) = \vec{0}$
6. $c\vec{u} \in V$
7. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
8. $(c + d)\vec{u} = c\vec{u} + d\vec{u}$
9. $c(d\vec{u}) = (cd)\vec{u}$
10. $1\vec{u} = \vec{u}$