From Lay, Section 1.2

DEFINITION

A rectangular matrix is in echelon form (or row echelon form) if it has the following three properties:

- 1. All nonzero rows are above any rows of all zeros.
- 2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
- 3. All entries in a column below a leading entry are zeros.

If a matrix in echelon form satisfies the following additional conditions, then it is in reduced echelon form (or reduced row echelon form):

- 4. The leading entry in each nonzero row is 1.
- **5.** Each leading 1 is the only nonzero entry in its column.

Echelon Form Reduced Echelon Form

$$\begin{bmatrix} 3 & -6 & 0 & 4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -6 & 0 & 4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1 0 3 0 3 5 is in echelon form 0 2 1

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Hmm...

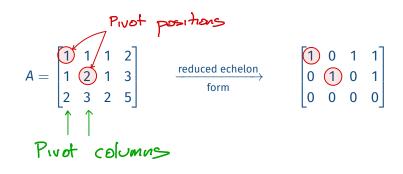
$$\begin{bmatrix} 1 & 0 & 0 & 1 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
 is in reduced echelon form

- (a) True, and I can explain why
- (b) True, but I am unsure why
- (c) False, and I can explain why
- (d) False, but I am unsure why
- (e) Hmm...

From Lay, Section 1.2

DEFINITION

A **pivot position** in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A. A **pivot column** is a column of A that contains a pivot position.



From Lay, Section 1.2

Variables that correspond to a pivot column are called **basic variables**, and variables that do *not* correspond to a pivot column are called **free variables**.

Augmented matrix from #6 on Tuesday:

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \end{bmatrix} \qquad \xrightarrow{\text{reduced echelon}} \qquad \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

 x_1 and x_2 are basic variables

 x_3 is a free variable

1. Find the general solutions of the system whose augmented matrix is

$$\begin{bmatrix} 2 & -8 & 0 & 1 & -1 & 4 \\ -4 & 16 & 3 & -2 & 17 & -14 \\ 6 & -24 & 0 & 5 & 3 & 16 \end{bmatrix}$$

2. Let
$$\vec{\mathbf{u}} = \langle 1, 2, -1 \rangle$$
, $\vec{\mathbf{v}} = \langle -3, 1, 5 \rangle$

(a) Does
$$\vec{\mathbf{w}} = \langle 7, 0, 2 \rangle$$
 lie in Span $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$?

(b) What does this tell you about the lines

$$x - 3y = 7$$
, $2x + y = 0$, and $-x + 5y = 2$?