From Lay, Section 1.2

DEFINITION

A rectangular matrix is in **echelon form** (or **row echelon form**) if it has the following three properties:

1. All nonzero rows are above any rows of all zeros.
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it.
3. All entries in a column below a leading entry are zeros.

If a matrix in echelon form satisfies the following additional conditions, then it is in **reduced echelon form** (or **reduced row echelon form**):

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only nonzero entry in its column.

Echelon Form

$$\begin{bmatrix} 3 & -6 & 0 & 4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reduced Echelon Form

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 3 & 5 \\ 0 & 2 & 1 \end{bmatrix} \text{ is in echelon form}$$

(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Hmm. . .

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ is in reduced echelon form

(a) True, and I can explain why

(b) True, but I am unsure why

(c) False, and I can explain why

(d) False, but I am unsure why

(e) Hmm. . .

From Lay, Section 1.2

DEFINITION

A **pivot position** in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A. A **pivot column** is a column of A that contains a pivot position.

Pivot positions

$$A = \begin{bmatrix} \textcircled{1} & 1 & 1 & 2 \\ 1 & \textcircled{2} & 1 & 3 \\ 2 & 3 & 2 & 5 \end{bmatrix} \xrightarrow[\text{form}]{\text{reduced echelon}} \begin{bmatrix} \textcircled{1} & 0 & 1 & 1 \\ 0 & \textcircled{1} & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot columns

From Lay, Section 1.2

Variables that correspond to a pivot column are called **basic variables**, and variables that do *not* correspond to a pivot column are called **free variables.**

Augmented matrix from #6 on Tuesday:

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \end{array}\right] \xrightarrow[\text{form}]{\text{reduced echelon}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_1 and x_2 are basic variables

x_3 is a free variable

1. Find the general solutions of the system whose augmented matrix is

$$\left[\begin{array}{rrrrr|r} 2 & -8 & 0 & 1 & -1 & 4 \\ -4 & 16 & 3 & -2 & 17 & -14 \\ 6 & -24 & 0 & 5 & 3 & 16 \end{array}\right]$$

2. Let $\vec{\mathbf{u}} = \langle 1, 2, -1 \rangle$, $\vec{\mathbf{v}} = \langle -3, 1, 5 \rangle$

(a) Does $\vec{\mathbf{w}} = \langle 7, 0, 2 \rangle$ lie in $\text{Span}\{\vec{\mathbf{u}}, \vec{\mathbf{v}}\}$?

(b) What does this tell you about the lines

$$x - 3y = 7, \quad 2x + y = 0, \quad \text{and} \quad -x + 5y = 2?$$