The Power of the Chair

Wheaton has (hypothetically) received a $750M unrestricted gift

A committee of 30 administrators, faculty, staff, students, and trustees is considering four options:

- A – Lower comprehensive fee across the board
- B – Renovate dorms
- C – Cover study abroad for every student
- D – Raise salaries of Math faculty

They are unanimous in their preferences: 30 $A > B > C > D$

The non-voting chair is a mathematician.

Can they get the committee to select D?

The Power of the Chair (cont.)

The chair introduces two new alternatives:

- E – Add an MBA program
- F – Make all athletics Division I

This polarizes the committee so that the preferences are now

10	$E > F > A > B > C > D$
10	$F > A > B > C > D > E$
10	$A > B > C > D > E > F$

Arrow's Theorem (1951)

Let F : Domain$\rightarrow$Range be a voting method where:

1. Each voter has a complete, strict transitive ranking.
2. There is no restriction on how the voters can rank the candidates.
3. The outcome is a transitive ranking, with ties allowed.
4. F satisfies the Pareto condition.
5. F satisfies Independence of Irrelevant Alternatives (IIA).

There is exactly one voting procedure that satisfies these five criteria: a dictatorship.

Theorem 11 (pg 123, *Chaotic Elections*, Saari, 2000)

Suppose that the Condorcet component is removed from all profiles. Then the Borda Count satisfies the assumptions of Arrow’s Theorem. This holds for any number of candidates.

Theorem 12 (pg 123 *Chaotic Elections*, Saari, 2000)

If we modify IIA to take into account the number of voters between A and B in a voter’s A vs B preference, then the Borda Count is a method that satisfies this modified version of Arrow’s Theorem.

No other positional method satisfies these conditions.