# Problem Set #5

Due Thursday, March 24, 2022 @ 11:59 pm
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*Remember that you need to explain and show the steps in your answers!*

1. Let $g(x) = \cos(2x) + \sin(2x)$

   (a) Verify that $g(x)$ has critical numbers at $x = \frac{5\pi}{8}$ and $x = \frac{9\pi}{8}$
   Note: These are not *all* of the critical numbers of $g(x)$, just two of them.

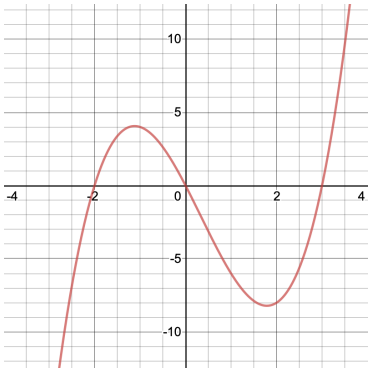
   (b) Use the Second Derivative Test to classify each critical number as a local min or local max, if possible.

2. The graph of $f''(x)$ shown below. This is *not* the graph of $f(x)$ or $f'(x)$!

   (a) Where is $f$ concave up? concave down?

   (b) Where does $f$ have inflection points?

   (c) Suppose that $f'(-1) = 0$ and $f'(1) = 0$.
   If possible, classify $x = -1$ and $x = 1$ as local maxima or local minima of $f$.



Plot of $y = f''(x)$

3. Use the graph of $y = f''(x)$ from #2 to answer the following.

   (a) Suppose that $f'(0) = 0$. Is $f$ increasing or decreasing at $x = 2$? at $x = -1$? Why?

   (b) Suppose that $f'(0) = -1$ and $f(0) = 1$. Is it possible that $f(2) = 3$? Explain.
   *Hint:* Can you determine if $f$ is increasing or decreasing on $[0, 2]$?

4. Evaluate the following limits.

   (a) $\lim_{x\to 0} \frac{\sin(3x)}{7x}$

   (b) $\lim_{x\to \infty} \frac{\ln(x)}{\sqrt{x}}$

   (c) $\lim_{x\to 0} \frac{\cos(x)}{x-1}$