Problem Set #3

Due Thursday, February 17, 2022 @ 11:59 pm
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Remember that you need to explain and show the steps in your answers!

1. Suppose that $f'(x) = 3\cos(x^2 - 1) + x^2 - x$
   (a) Use the Intermediate Value Theorem to show that $f'(x)$ has a root between $x = 2$ and $x = 3$.
   (b) Will this root of $f'(x)$ be a local maximum or a local minimum of $f(x)$? Explain.

2. Let $g(x) = \dfrac{2x^2 - 6x - 8}{5x^2 - 25x + 20}$
   (a) Find $\lim_{x \to 1^+} g(x)$
   (b) Find $\lim_{x \to 4} g(x)$
   (c) Find $\lim_{x \to \infty} g(x)$

3. Use function $g(x)$ from #2 and your answers to #2 to answer the following.
   (a) Does $g(x)$ have any vertical asymptotes? If so, where?
   (b) Does $g(x)$ have any horizontal asymptotes? If so, where?
   (c) Is $g(x)$ continuous at $x = 4$? Explain.

4. Let $f(x) = x^2 - 5x + 2$
   (a) Use the limit definition of the derivative to find $f'(1)$
   (b) Use your answer to part (a) to find the equation of the line tangent to the graph of $y = f(x)$ at $x = 1$
   (c) Verify your answer by graphing $y = f(x)$ and your line from part (b) on the same set of axes. Include your graph with your solutions.