Why the product rule is true:

$$\frac{d}{dx}\left(f(x)\,g(x)\right)$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)\,g(x+h) - f(x)\,g(x+h)}{h} + \frac{f(x)\,g(x+h) - f(x)\,g(x)}{h}\right)$$

$$= \lim_{h\to 0} \left(\left[\frac{f(x+h) - f(x)}{h}\right] g(x+h) \;+\; f(x)\left[\frac{g(x+h) - g(x)}{h}\right]\right)$$

$$= f'(x)g(x) + f(x)g'(x)$$

Why the quotient rule is true:

Let $h(x) = \dfrac{f(x)}{g(x)}$. We want to find $h'(x)$

$$f(x) = g(x)h(x)$$

$$\Rightarrow \quad f' = g'h + gh' \qquad \text{by the Product Rule}$$

$$f' = g'\left(\frac{f}{g}\right) + gh'$$

$$f'g = g'f + g^2h'$$

$$f'g - g'f = g^2h'$$

$$h' = \frac{f'g - g'f}{g^2}$$

1. Find the derivative of each function
   (a) $h(x) = x^{32}\ln(x)$
   (b) $h(x) = \sqrt{x}\left(\ln(x) + \frac{2}{x^3}\right)$
   (c) $h(x) = \frac{3 + 2x^{-3}}{8x^3 - 4x}$
   (d) $h(x) = x\ln(x) - x$
2. Find an antiderivative for each function
   (a) $f(x) = 2x\sin(x) + x^2\cos(x)$
   (b) $f(x) = \ln(x) + \frac{1}{x}$