

1. Let $f(x) = \sin(x) + 6\sqrt[3]{x} - \frac{1}{x} + 3e^x$

(a) Find $f'(x)$ and $f''(x)$

(b) Find an *antiderivative* of $f(x)$. That is, a function $F(x)$ where $F'(x) = f(x)$

2. Let $g(x) = 2 \cos(x) - \ln(x)$

(a) Find $g'(x)$ and $g''(x)$

(b) Apply the IVT to $g'(x)$ to show that $g'(x)$ has a root between $x = 2$ and $x = 4$

(c) Use your answer to (b) to show that $g(x)$ has a local minimum between $x = 2$ and $x = 4$

3. How can you find the value of $\sin(3)$?

If we can find the line tangent to $y = \sin(x)$ at a point near $x = 3$, we can use this line to approximate $\sin(3)$ since $f(x) = \sin(x)$ is locally linear

(a) Find the equation of the line tangent to $y = \sin(x)$ at $x = \pi$

(b) Use your tangent line to approximate $\sin(3)$

(c) Use your tangent line to approximate $\sin(7)$.

Is this a good approximation?

4. Use that $g(x) = \cos(x)$ is locally linear to approximate $\cos(4)$

Hint: Find a value x_0 close to $x = 4$ where you know both $g(x_0)$ and $g'(x_0)$