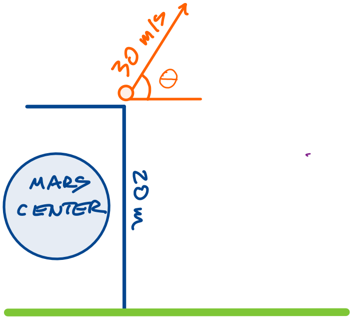
#4 from yesterday: Find angle that maximizes distance



We want to maximize horizontal distance traveled

h = (horizontal velocity) · (time when hits ground)

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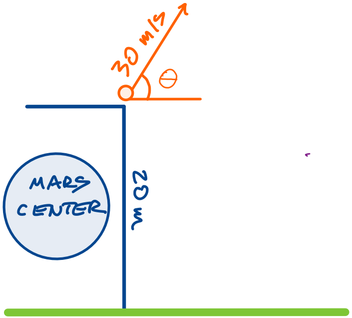

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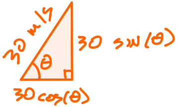
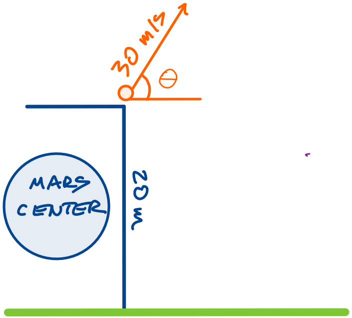
We want to write as a function of θ



horizontal velocity $= 30\cos(\theta)$ m/s



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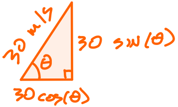


horizontal velocity $= 30\cos(\theta)$ m/s

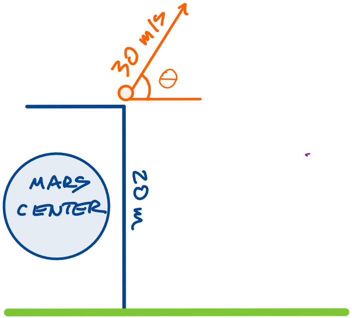
Find time when hits ground

Vertical Position

$$p(t) = p_0 + v_0 t + \frac{1}{2} a t^2$$

$$= 20 + 30\sin(\theta)\, t - 4.9\, t^2$$

$$= 0$$



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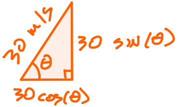

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Find time when hits ground

$$p(t) = p_0 + v_0 t + \frac{1}{2} a t^2$$

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Quadratic formula gives positive root

$$t = \frac{30\sin(\theta)\, t + \sqrt{900\sin(\theta)^2 + 392}}{9.8} \text{ s}$$

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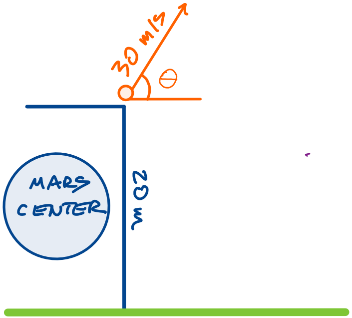

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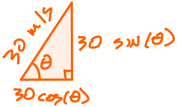

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Quadratic formula gives positive root

$$t = \frac{30\sin(\theta)t + \sqrt{900\sin(\theta)^2 + 392}}{9.8} \text{ s}$$

We want to maximize

$$h(\theta) = (30\cos(\theta) \text{ m/s})\left(\frac{30\sin(\theta)t + \sqrt{900\sin(\theta)^2 + 392}}{9.8} \text{ s}\right) \quad \text{on } [-\pi/2, \pi/2]$$

We want to maximize

$$h(\theta) = \frac{30}{9.8} \cdot \cos(\theta) \left(30 \sin(\theta) + \sqrt{900 \sin^2(\theta) + 392} \right) \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

We want to maximize

$$h(\theta) = \underbrace{\frac{30}{9.8} \cdot \cos(\theta)}_{u} \underbrace{\left(30\sin(\theta) + \sqrt{900\sin^2(\theta) + 392}\right)}_{v} \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Product Rule!

$$h(\theta) = \frac{30}{9.8} \cdot \cos(\theta)\left(30\sin(\theta) + \sqrt{900\sin^2(\theta) + 392}\right) \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

u v

Find the critical points on the interval:

u v'

$$h'(\theta) = \frac{30}{9.8}\cos(\theta)\left(30\cos(\theta) + \frac{900\sin(\theta)\cos(\theta)}{\sqrt{900\sin^2(\theta)+392}}\right)$$

$$+ \quad -\frac{30}{9.8}\sin(\theta)\left(\sqrt{900\sin^2(\theta)+392} + 30\sin(\theta)\right)$$

u' v

We want to maximize

$$h(\theta) = \frac{30}{9.8} \cdot \cos(\theta)\left(30\sin(\theta) + \sqrt{900\sin^2(\theta) + 392}\right) \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Find the critical points on the interval:

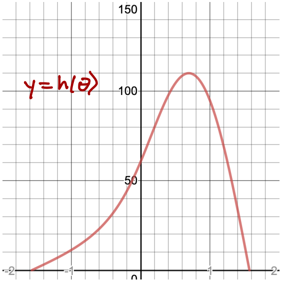
$$h'(\theta) = \frac{30}{9.8}\cos(\theta)\left(30\cos(\theta) + \frac{900\sin(\theta)\cos(\theta)}{\sqrt{900\sin^2(\theta)+392}}\right)$$

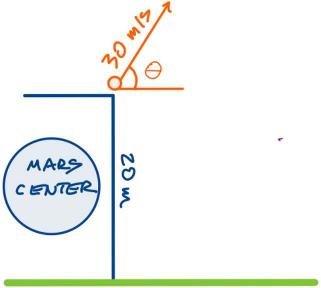
$$- \frac{30}{9.8}\sin(\theta)\left(\sqrt{900\sin^2(\theta)+392} + 30\sin(\theta)\right)$$

$$h'(\theta) = 0 \quad \Rightarrow \quad \theta = 0.695499 \text{ radians } = 39.84^\circ$$

Check values at endpoints and critical points

θ	$h(\theta)$
$-\dfrac{\pi}{2}$	0
0.695499	110.034 m
$+\dfrac{\pi}{2}$	0

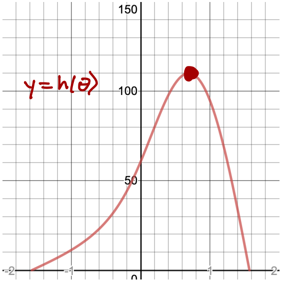


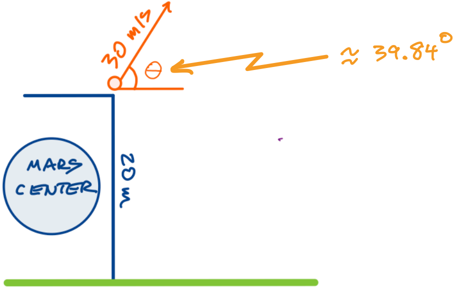


Check values at endpoints and critical points

θ	$h(\theta)$
$-\frac{\pi}{2}$	0
0.695499	110.034 m
$+\frac{\pi}{2}$	0

Max





Let $\mathcal{I} = \int_0^1 e^{(x^3)}\, dx$

1. Approximate $\mathcal{I}$ using L_{50}, a left sum with 50 subdivisions

2. Use a Maclaurin polynomial to approximate $\mathcal{I}$

3. How close are your answers?