Quick recap on Series

- *n*th Term Test: If $\lim_{k\to\infty} a_k \neq 0$, then $\sum_{k=0}^{\infty} a_k$ diverges

- Geometric Series Test $\sum_{k=0}^{\infty} r^k$
  - If $|r| \geq 1$, then the series diverges
  - If $|r| < 1$, then the series converges to $\dfrac{1}{1-r}$

- Integral Test: If $a(x) > 0$, decreasing, and $a_k = a(k)$, then

$$\sum_{k=1}^{\infty} a_k \text{ and } \int_1^{\infty} a(x)\, dx$$

  behave exactly the same. They either both converge or both diverge

Quick recap on Series (cont.)

- **The p-test** $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^p}$
  - If $|p| \leq 1$, then the series diverges
  - If $p > 1$, then the series converges

- **Direct Comparison Test:** If $0 \leq a_k \leq b_k$ then $0 \leq \displaystyle\sum_{k=1}^{\infty} a_k \leq \sum_{k=1}^{\infty} b_k$ and
  - If $\displaystyle\sum_{k=1}^{\infty} b_k$ converges, then $\displaystyle\sum_{k=1}^{\infty} a_k$ also converges
  - If $\displaystyle\sum_{k=1}^{\infty} a_k$ diverges, then $\displaystyle\sum_{k=1}^{\infty} b_k$ also diverges

Quick recap on Series (cont.)

- Integral, p, and Direct Comparison tests apply only to positive series
- nth Term, Integral, p, and Direct Comparison determine convergence or divergence
  - They do *not* give limit of convergent series
  - Can approximate value by calculating a large partial sum, like S_{100}
- Geometric Series Test determines convergence, divergence, *and* value of convergent geometric series

Do the following series converge or diverge?

If a series converges, find the exact value to which it converges, if possible
If you cannot find the exact value, approximate it by using WolframAlpha to compute S_{100}

1. $\displaystyle\sum_{k=0}^{\infty} \frac{-3}{5^k}$
2. $\displaystyle\sum_{k=1}^{\infty} \frac{3k^2+1}{2k^2+k+2}$
3. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$
4. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{4^k+7}$
5. $\displaystyle\sum_{k=42}^{\infty} \frac{7^k}{5^k-k}$
6. $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k\ \ln(k)}$