Recap on Series from Monday

- There are two sequences associated with every series $\sum_{k=1}^{\infty} a_k$
  - The sequence of terms $\{a_k\}$ and
  - The sequence of partial sums $\{S_n\}$ where $S_n = a_1 + \cdots + a_n = \sum_{k=1}^{n} a_k$
  - The series converges if the sequence of partial sums converges

- A geometric series is of the form $\sum_{k=0}^{\infty} r^k = 1 + r + r^2 + r^3 + \cdots$
  - If $|r| < 1$, then the series converges to $\frac{1}{1-r}$.
  - If $|r| \geq 1$, then the series diverges.

For each series

(a) Write the first five terms of the series
Does the sequence of terms converge? If so, what is the limit?

(b) Write the first five partial sums of the series
Does the sequence of partial sums converge? If so, what is the limit?

(c) Does the series converge? If so, what is the limit?

1. $\displaystyle\sum_{k=0}^{\infty}\left(-\frac{1}{3}\right)^k$

2. $\displaystyle\sum_{k=0}^{\infty}\left(\frac{5}{3}\right)^k$

3. $\displaystyle\sum_{k=0}^{\infty}\frac{2}{5^k}$

4. $\displaystyle\sum_{k=0}^{\infty}\frac{6}{2^k}$

5. $\displaystyle\sum_{k=2}^{\infty}\left(\frac{2}{3}\right)^k$

6. $\displaystyle\sum_{k=17}^{\infty}\left(\frac{4}{5}\right)^k$

7. $\displaystyle\sum_{k=0}^{\infty}\frac{k}{k+1}$

8. $\displaystyle\sum_{k=1}^{\infty}\left(2-\frac{1}{k}\right)$

The nth Term Test

If $\lim\limits_{k\to\infty} a_k \neq 0$, then $\sum\limits_{k=0}^{\infty} a_k$ diverges

Notice:

- The nth Term Test can only tell us that the series *diverges*
- It cannot tell us that a series converges
- If $\lim\limits_{k\to\infty} a_k = 0$, then the nth Term Test tells us nothing