Recap on Improper Integrals from Monday

- $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx = 1$

- $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^3}\,dx = \frac{1}{2}$

- $\displaystyle\int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx$ diverges

- $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{\sqrt{x}}\,dx = \cdots$

Explain why each integral is improper and determine if the integral converges or diverges

1. $\displaystyle\int_1^\infty \frac{1}{x^4}\,dx$

2. $\displaystyle\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$

3. $\displaystyle\int_1^\infty \frac{1}{x^2+1}\,dx$

4. $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$

5. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$

6. $\displaystyle\int_0^1 \frac{1}{x}\,dx$

The p-Test for Improper Integrals

$$\int_1^\infty \frac{1}{x^p}\,dx$$

- converges if $p > 1$
  In antiderivative, x stays in denominator $\Rightarrow \lim_{b\to\infty}$ exists
- diverges if $p = 1$
  Antiderivative is $\ln(x) \Rightarrow \lim_{b\to\infty}$ DNE
- diverges if $p < 1$
  In antiderivative, x moves to numerator $\Rightarrow \lim_{b\to\infty}$ DNE

$$\int_0^1 \frac{1}{x^p}\,dx$$

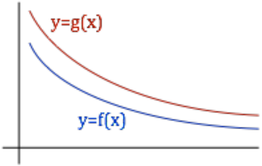
- diverges if $p > 1$
  In antiderivative, x stays in denominator $\Rightarrow \lim_{b\to 0}$ DNE
- diverges if $p = 1$
  Antiderivative is $\ln(x) \Rightarrow \lim_{b\to 0}$ DNE
- converges if $p < 1$
  In antiderivative, x moves to numerator $\Rightarrow \lim_{b\to 0}$ exists

Example

$$\int_1^\infty \frac{1}{x^8 + 1}\, dx$$

Direct Comparison Test for Improper Integrals

Suppose $0 \leq f(x) \leq g(x)$ on the interval $[1, \infty)$



$$\text{Then } 0 \leq \int_1^\infty f(x)\, dx \leq \int_1^\infty g(x)\, dx$$

- If $\int_1^\infty g(x)\, dx$ converges, then $\int_1^\infty f(x)\, dx$ also converges
- If $\int_1^\infty f(x)\, dx$ diverges, then $\int_1^\infty g(x)\, dx$ also diverges