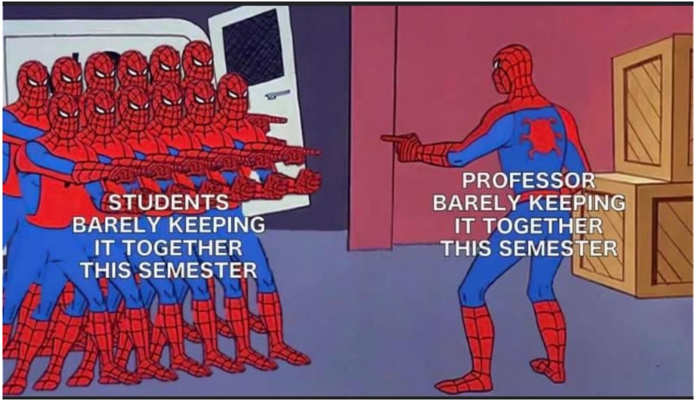
Announcements

- Same schedule for tutorials as last week

- Problem Set due Friday



Quick overview of directional derivatives

- The gradient of $f(x, y)$ is the vector-valued function

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle$$

Quick overview of directional derivatives

- The gradient of $f(x, y)$ is the vector-valued function

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle$$

- If $\vec{\mathbf{u}}$ is a unit vector, then we calculate the directional derivative of f at (x_0, y_0) in the $\vec{\mathbf{u}}$ direction by

$$D_{\vec{\mathbf{u}}} f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \vec{\mathbf{u}}$$

- Notice $\vec{\mathbf{u}}$ must be a unit vector!

Let $f(x, y) = 3xy^2 + 2x - 4y^2$ and $\vec{u} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

Then $D_{\vec{\mathbf{u}}} f(2, 1)$, the directional derivative of f at $(2, 1)$ in the direction of $\vec{u}$, is

(a) $\langle 5, 4 \rangle$

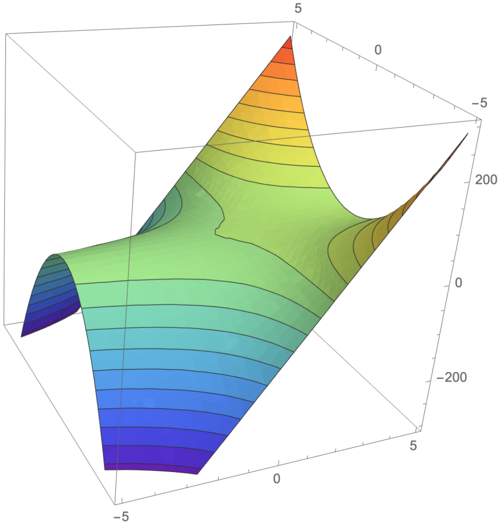
(b) $\left\langle 3, \frac{16}{5} \right\rangle$

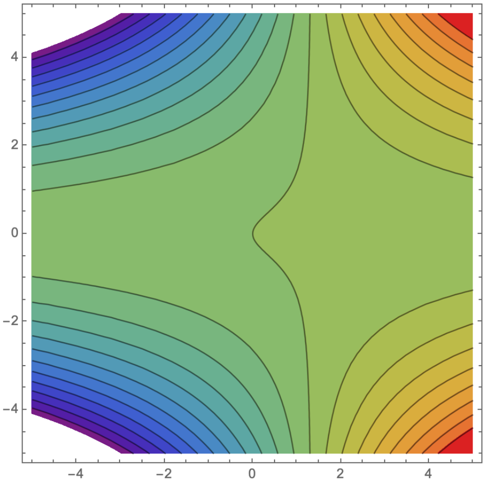
(c) $-\frac{1}{5}$

(d) $\frac{31}{5}$

(e) I'm not willing to commit. . .

$f(x, y) = 3xy^2 + 2x - 4y^2$





Let $f(x, y) = 3xy^2 + 2x - 4y^2$ and $\vec{v} = \langle -3, -1 \rangle$

Then $D_{\vec{\mathbf{u}}} f(2, 1)$, the directional derivative of f at $(2, 1)$ in the direction of $\vec{v}$, is

(a) -19

(b) $\left\langle -\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \right\rangle$

(c) -6.008

(d) $-\frac{19}{\sqrt{10}}$

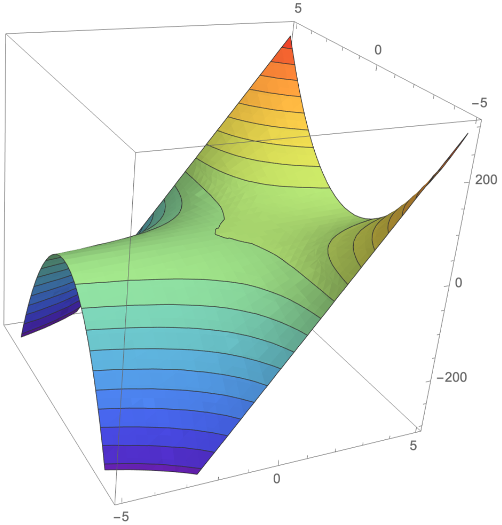
(e) I'm not willing to commit. . .

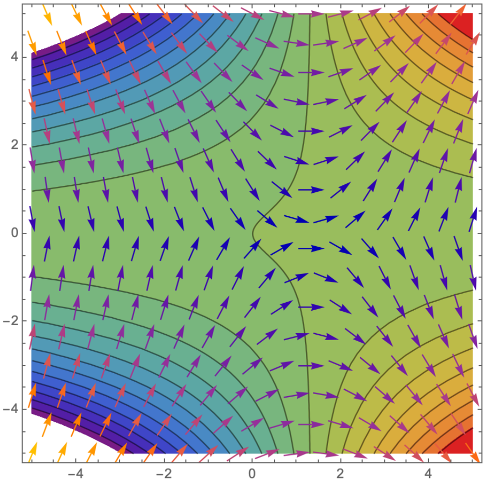
Consequences of $D_{\vec{\mathbf{u}}} f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \vec{\mathbf{u}} = \|\nabla f(x_0, y_0)\| \cos(\theta)$

At a point (x_0, y_0),

- f increases fastest in the direction of $\nabla f(x_0, y_0)$
- f decreases fastest in the direction of $-\nabla f(x_0, y_0)$
- $\nabla f(x_0, y_0)$ is perpendicular to the level curve

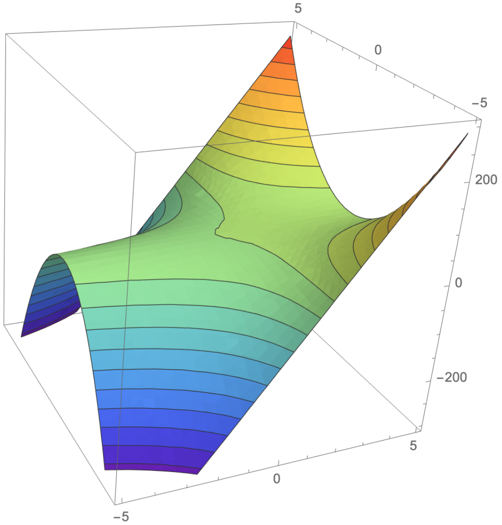
$f(x, y) = 3xy^2 + 2x - 4y^2$ increases fastest in the direction of ∇f

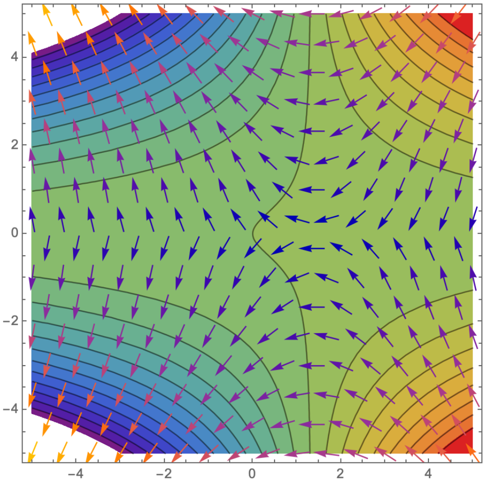




∇f overlayed on contour plot of f

$f(x, y) = 3xy^2 + 2x - 4y^2$ decreases fastest in the direction of $-\nabla f$





$-\nabla f$ overlayed on contour plot of f

Cool application of ∇f being perpendicular to level curves

One of ideas behind converting a digital image to look like a painting

- https://www.vfxvoice.com/painting-the-afterlife-in-what-dreams-may-come/
- https://revisionfx.com/company/
- https://vimeo.com/144021430