

# Announcements

- Exam 2 this week - Covers Improper Integrals and Series
  - Cheat Sheet due Wednesday @ 8:00 am
  - Exam available Wednesday @ 8:00 am
  - Due Friday @ midnight
  - Same ground rules as for Exam 1
- “Normal” schedule for tutorials this week
- Let me know if you have any questions about courses for next semester

# Basics of vectors

- If  $\vec{\mathbf{a}} = \langle a_1, a_2 \rangle$  then the length of  $\vec{\mathbf{a}}$  is  $\|\vec{\mathbf{a}}\| = \sqrt{a_1^2 + a_2^2}$
- The unit vector pointing in the same direction as  $\vec{\mathbf{a}}$  is

$$\vec{\mathbf{u}} = \frac{\vec{\mathbf{a}}}{\|\vec{\mathbf{a}}\|} = \frac{1}{\|\vec{\mathbf{a}}\|} \langle a_1, a_2 \rangle$$

- If  $\vec{\mathbf{a}} = \langle a_1, a_2 \rangle$  and  $\vec{\mathbf{b}} = \langle b_1, b_2 \rangle$ , then the *dot product* is defined by

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1 b_1 + a_2 b_2$$

**Let  $\vec{v}_1 = \langle 2, 3 \rangle$ ,  $\vec{v}_2 = \langle -6, 4 \rangle$**

## Key result

If  $\theta$  is the angle between  $\vec{\mathbf{a}}$  and  $\vec{\mathbf{b}}$  then  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \|\vec{\mathbf{a}}\| \|\vec{\mathbf{b}}\| \cos(\theta)$

- Shows  $\vec{\mathbf{a}}$  and  $\vec{\mathbf{b}}$  are perpendicular if and only if  $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$
- Allows us find angle between *any* two vectors:  $\cos(\theta) = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{\|\vec{\mathbf{a}}\| \|\vec{\mathbf{b}}\|}$
- Also carries over to vectors in 3-space

Let  $\vec{\mathbf{a}} = \langle 2, -1 \rangle$  and  $\vec{\mathbf{b}} = \langle -1, 4 \rangle$

- Find  $\vec{\mathbf{u}}_{\mathbf{a}}$ , the unit vector in the same direction as  $\vec{\mathbf{a}}$
- Find  $\vec{\mathbf{u}}_{\mathbf{b}}$

Let  $\vec{\mathbf{a}} = \langle 2, -1 \rangle$  and  $\vec{\mathbf{b}} = \langle -1, 4 \rangle$

- Find the angle between  $\vec{\mathbf{a}}$  and  $\vec{\mathbf{b}}$
- Find the angle between  $\vec{\mathbf{u}}_{\mathbf{a}}$  and  $\vec{\mathbf{u}}_{\mathbf{b}}$

Let  $\vec{a} = \langle 2, -1 \rangle$

Give two vectors perpendicular to  $\vec{a}$

Let  $\vec{a} = \langle 1, -3, 2 \rangle$  and  $\vec{b} = \langle 3, 0, 1 \rangle$

- Find  $\vec{u}_a$ , the unit vector in the same direction as  $\vec{a}$
- Find the angle between  $\vec{a}$  and  $\vec{b}$
- Find two vectors perpendicular to  $\vec{a}$