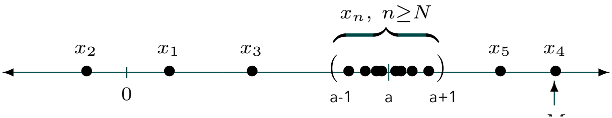
Theorem 2.3.2: Every convergent sequence is bounded.

1. For each sequence from Thursday's in-class work, find a bound on the sequence. That is, find an $M \in \mathbb{R}$ such that $|a_n| < M \ \forall \ n \in \mathbb{N}$.

2. Fill in the details of the proof of the Theorem.
   - Suppose that (x_n) converges to a
     We want to show $\exists M \in \mathbb{R}$ such that $|x_n| \leq M$ for all n
   - Pick $\epsilon = 1$. Then there exists N such that $x_n \in V_1(a)$ for all $n \geq N$ (Why?)
   - Then $|x_n| < |a| + 1$ for all $n \geq N$ (Why? What if $a < 0$?)



   - Pick M accordingly
   - State the conclusion (And draw a box. And check it.)

Theorem 2.3.3 (Algebraic Limit Theorem)

Let $\lim a_n = a$ and $\lim b_n = b$. Then

1. $\lim(ca_n) = ca$ for all c$\in \mathbb{R}$
2. $\lim(a_n + b_n) = a + b$
3. $\lim(a_n b_n) = ab$
4. $\lim \left(\frac{a_n}{b_n}\right) = \frac{a}{b}$ if $b \neq 0$

Theorem 2.3.4 (Order Limit Theorem)

Let $\lim a_n = a$ and $\lim b_n = b$. Then

1. If $a_n \geq 0$ for all $n \in \mathbb{N}$, then $a \geq 0$
2. If $a_n \leq b_n$ for all $n \in \mathbb{N}$, then $a \leq b$
3. If there exists $c \in \mathbb{R}$ such that $c \leq b_n$ for all $n \in \mathbb{N}$, then $c \leq b$
   Similarly, if $a_n \leq c$ for all $n \in \mathbb{N}$, then $a \leq c$