

## Theorem 1.5.7: If $A \subset B$ and $B$ is countable, then $A$ is either countable or finite.

Fill in the details of the proof:

- If  $A$  is finite, then we are done.
- Suppose  $A$  is infinite. We want to define one-one onto  $g : \mathbb{N} \rightarrow A$ 
  - Let  $f : \mathbb{N} \rightarrow B$  be one-one onto  $b_1, b_2, b_3, \dots$
  - Let  $n_1 = \min \{n \in \mathbb{N} \mid f(n) \in A\}$ . i.e. Find first element of  $A$  in the listing of  $B$   
Define  $g(1) = f(n_1)$
  - Let  $n_2 = \min \{n \in \mathbb{N} \mid f(n) \in A - \{g(1)\}\}$ . i.e. Find next element of  $A$  in the list  
Define  $g(2) = f(n_2)$
  - Use the same process to define the general terms  $n_i$  and  $g(i)$
  - Use that  $f$  is one-one and onto to show that  $g$  is one-one onto