

1. Construct a one-one and onto function  $f: (0, 1) \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$  to show  $(0, 1) \sim (-\frac{\pi}{2}, \frac{\pi}{2})$
2. Show  $(0, 1) \sim (a, b)$  for all  $a, b \in \mathbb{R}$  such that  $a < b$
3. Show  $(-\frac{\pi}{2}, \frac{\pi}{2}) \sim \mathbb{R}$       *Hint: Think about a trig function*
- \*3. Let  $\mathbb{Q}^+$  be the set of positive rational numbers. Use the following to show  $\mathbb{Q}^+ \sim \mathbb{N}$ 
  - (a) Make a table of all positive rational numbers so that each fraction  $\frac{p}{q}$  appears in the  $p$ th column and the  $q$ th row. (Okay, just go out as far as  $p, q = 5$ .)
  - (b) Cross out duplicates that are not in lowest terms.
  - (c) Turn your table  $45^\circ$  clockwise, so that  $\frac{1}{1}$  is in the top “row”. There should be two numbers in the next row, and more numbers as you move further down. Reading this twisted table, list in order the first dozen numbers you encounter.
  - (d) Conclude that  $\mathbb{Q}^+$  is countable.
4. Show that  $\mathbb{Q}$  is countable.

*\* Shamelessly stolen, with permission, from Annalisa Crannell at Franklin & Marshall*