1. Fill in the details of the proof of the Archimedean Property

(i) Given any $x \in \mathbb{R}$, there exists $n \in \mathbb{N}$ such that $n > x$.

- Use a proof by contradiction
- Assume $\exists\, x \in \mathbb{R} \ni n \leq x \;\forall\, n \in \mathbb{N}$
- Then $\exists\, \alpha \in \mathbb{R}$ where $\alpha = \sup \mathbb{N}$
- Then $\exists\, n \in \mathbb{N}$ such that $\alpha - 1 < n$
- Then $\alpha < n + 1$
- Oops.

(ii) Given any real number $y > 0$, there exists $n \in \mathbb{N}$ such that $\frac{1}{n} < y$.

- Let $x = \frac{1}{y}$ and apply part (i)

2. Density of $\mathbb{Q}$ in $\mathbb{R}$

1. Find $r = \frac{m}{n} \in \mathbb{Q}$ such that $\sqrt{2} < r < \sqrt{3}$
2. Find $r = \frac{m}{n} \in \mathbb{Q}$ such that $\frac{333}{106} < r < \frac{22}{7}$
3. Find a value for the denominator n that guarantees there is a rational $r = \frac{m}{n} \in \mathbb{Q}$ such that $\sqrt{2} < r < \sqrt{3}$
4. For arbitrary $a, b \in \mathbb{R}$, find a value for the denominator n that guarantees there is a rational $r = \frac{m}{n} \in \mathbb{Q}$ such that $a < r < b$