For each function, sketch a graph.
Where do you think the function is continuous? Discontinuous?

1. $f(x) = 2x + 3$
2. $g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$
3. $h(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$
4. $t(x) = \begin{cases} 1 & \text{if } x = 0 \\ \frac{1}{n} & \text{if } x = \frac{m}{n} \in \mathbb{Q} - \{0\} \text{ in lowest terms } n > 0 \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$

   Hint: First consider $x \in \mathbb{Z}$, then $x = \frac{k}{2}, k \in \mathbb{Z}$, then $x = \frac{k}{3}, k \in \mathbb{Z}$, etc

Theorem 4.2.3 (Sequential Criterion for Functional Limits)
Let $f : A \to \mathbb{R}$ and let c be a limit point of A.

Then $\lim_{x \to c} f(x) = L$ iff for all sequences (x_n) in A satisfying $x_n \neq c$ and $(x_n) \to c$ it follows that $f(x_n) \to L$.

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Corollary 4.2.5 (Divergence Criterion for Functional Limits)
Let $f : A \to \mathbb{R}$ and let c be a limit point of A.

If there exist two sequences (x_n) and (y_n) in A where $x_n \neq c$ and $y_n \neq c$ and

$$\lim x_n = \lim y_n = c \text{ but } \lim f(x_n) \neq \lim f(y_n)$$

then the limit $\lim_{x \to c} f(x)$ does not exist.