

Answer each question as True or False and give some justification

1. If (b_n) is a sequence of positive real numbers, then the partial sums for the series $\sum_{n=1}^{\infty} b_n$ form a bounded sequence.
2. If (b_n) is a sequence of positive real numbers, then the partial sums for the series $\sum_{n=1}^{\infty} b_n$ form a monotone sequence.

3. The telescoping series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

converges.

4. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

(Hint: Consider the series in #3 and the Monotone Convergence Theorem).

5. Give a statement of the Cauchy Criterion applied to the partial sums of a series

$$\sum_{n=1}^{\infty} b_n$$

6. Show that a series $\sum b_n$ converges iff for all $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that whenever $n > m \geq N$, it follows that $|b_{m+1} + b_{m+2} + \cdots + b_n| < \epsilon$

7. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n}$

(a) Calculate the following partial sums: s_1, s_2, s_4, s_8

(b) Show $|s_2 - s_1| \geq \frac{1}{2}$, $|s_4 - s_2| \geq \frac{1}{2}$, and $|s_8 - s_4| \geq \frac{1}{2}$

(c) Show that if $j \geq 1$, then $|s_{2^{j+1}} - s_{2^j}| \geq \frac{1}{2}$

(d) Use #6 to conclude that the series diverges.

Shamelessly motivated by/stolen from, with permission, Annalisa Crannell at Franklin & Marshall