1. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 - 2$. Argue that f is continuous on $\mathbb{R}$.

2. Show how the Intermediate Value Theorem can be applied to f to show the existence of $\sqrt{2} \in \mathbb{R}$.

3. Let $g : \mathbb{Q} \to \mathbb{Q}$ by $g(x) = x^2 - 2$. Argue that g is continuous on $\mathbb{Q}$.

4. Show that the Intermediate Value Theorem does not hold for g.

This hints that we'll need to use the Axiom of Completeness to prove the IVT.